Complex number 2

1. Given $z_1 = 2 + i$, $z_2 = 3 - 4i$ and $\frac{1}{z_3} = \frac{1}{z_1} + 2z_2$, find z_3.

   Write your answer in the standard form of $a + bi$.
   Hence, find the modulus and principal argument of z_3.
   State your answers correct to three decimal places.

2. Given that $|z - 1| = 2|z + 1|$, find the Cartesian equation of the locus of the point P representing complex number z. .
   Hence, sketch the locus of the point P on an Argand diagram.

3. Solve the equation $z^5 + 32i = 0$.

4. If the equation $z^3 + az + b = 0$ has a root $z = -1 + i$ where a, b are real numbers, find the values of a, b. Show that $z = -1 - i$ is also a root of the equation.

5. **(a)** One of the roots of the equation $4x^3 + x + 5 = 0$ is an integer. Find this root and write down a quadratic equation for the remaining roots. Find these roots, expressing your answer in the satandard form of $a + bi$.

   (b) By writing $y = \frac{1}{x}$, find the roots of the equation $5y^3 + y^2 + 4 = 0$,

   giving the complex roots in the form $a + bi$.

6. Find the roots of the equation $(z - i\alpha)^3 = i^3$, where α is a real constant.
   (a) Show that the points representing the roots of the above equation form an equilateral triangle.
   (b) Solve the equation $[z - (1 + i)]^3 = (2i)^3$.
   (c) If ω is a root of the equation $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a \neq 0$, show that the conjugate ω' is also a root of this equation.
   (d) Hence, or otherwise, obtain a polynomial equation of degree six with three of its roots also the roots of the equation $(z - 1)^3 = i^3$